TRINITY COLLEGE

Semester Two Examination, 2020

Question/Answer booklet

MATHEMATICS SPECIALIST UNITS 1&2

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures ☐☐ ☐☐☐ ☐☐☐

In words ______________________________

Your name ______________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): ☐

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

(a) Triangle ABC has vertices $A(2,-3), B(2,5)$ and $C(12,-1)$. Determine the area of this triangle after it has been transformed using the matrix $\begin{bmatrix} -4 & 4 \\ 3 & 3 \end{bmatrix}$. (3 marks)

Solution
Area of $\Delta ABC = \frac{1}{2} \times 8 \times 10 = 40$. Determinant of transformation matrix $= -24$. Area of transformed triangle $= \lvert -24 \rvert \times 40 = 960$ square units.
Specific behaviours
✓ area of ΔABC ✓ correct use of determinant ✓ correct area

(b) Show use of matrix algebra, including the coefficients of any inverse matrix used, to solve the following system of linear equations: (3 marks)

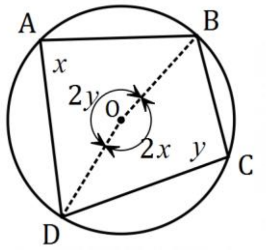
$$2a + 3b = 55$$
$$4a + 5b = 79$$

Solution
$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 55 \\ 79 \end{bmatrix}$ $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 55 \\ 79 \end{bmatrix}$ $= \frac{1}{-2} \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 55 \\ 79 \end{bmatrix} \left(\text{or uses } \begin{bmatrix} -\frac{5}{2} & \frac{3}{2} \\ 2 & -1 \end{bmatrix} \right)$ $= \begin{bmatrix} -19 \\ 31 \end{bmatrix}$
Specific behaviours
✓ writes system in matrix form ✓ matrix expression for solution, including inverse ✓ correct solution

Question 10 **(6 marks)**

(a) Prove that the opposite angles of a cyclic quadrilateral are supplementary. (3 marks)

Solution



Required to prove that $x + y = 180°$

$B\hat{O}D = 2 \times B\hat{A}D = 2x$ (Angle at centre-circumference)

$B\check{O}D = 2 \times B\hat{C}D = 2y$ (Angle at centre-circumference)

$2x + 2y = 360°$ (Angle at a point)

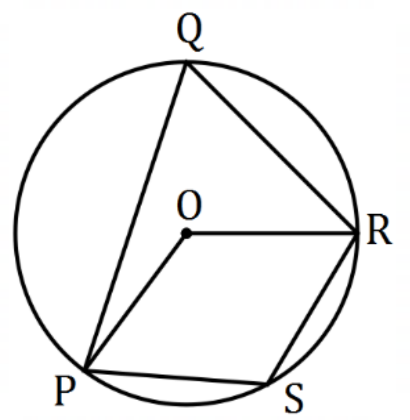
Hence $x + y = 180°$ as required.

Specific behaviours

✓ diagram and states required to prove
✓ uses angles at centre and on circumference
✓ completes proof

(b) The points P, Q, R and S lie on the circle with centre O so that $PS = RS$ and $\angle PQR = 48°$.

Determine the size of $\angle ORS$. (3 marks)



Solution

$\angle PSR = 180° - 48° = 132°$

$\Delta ROS \equiv \Delta POS$ (SSS)

$\angle OSR = 132° \div 2 = 66°$

$\angle ORS = \angle OSR = 66°$

Specific behaviours

✓ states an angle in $PORS$
✓ indicates property of $PORS$ (kite, congruency, etc)
✓ correct angle

Question 11 **(8 marks)**

Two vectors are $\mathbf{p} = \begin{pmatrix} 72 \\ -154 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -39 \\ 252 \end{pmatrix}$. Determine

(a) the magnitude of $\mathbf{p}$. (1 mark)

Solution
$\sqrt{72^2 + 154^2} = 170$
Specific behaviours
✓ correct magnitude

(b) the angle between the directions of $\mathbf{q}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. (2 marks)

Solution
$\cos^{-1}\dfrac{252}{255} = 8.8°$ (or using CAS)
Specific behaviours
✓ indicates correct method ✓ correct angle to nearest degree

(c) the value of the scalar constant k so that $18\mathbf{p} + k\mathbf{q}$ is parallel to $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. (2 marks)

Solution
$18\begin{pmatrix} 72 \\ -152 \end{pmatrix} + k\begin{pmatrix} -39 \\ 252 \end{pmatrix} = a\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $18(-152) + 252k = 0$ $k = 11$
Specific behaviours
✓ equation with k ✓ value of k

(d) a vector $\mathbf{r}$ that is perpendicular to $\mathbf{p}$ with the magnitude of $\mathbf{q}$. (3 marks)

Solution
$\mathbf{r} = a\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 72 \\ -154 \end{pmatrix} = a\begin{pmatrix} 154 \\ 72 \end{pmatrix}$ $\lvert a \rvert = \dfrac{255}{170} \Rightarrow a = \pm 1.5$ $\mathbf{r} = \begin{pmatrix} 231 \\ 108 \end{pmatrix}\ \left(\text{or } \boldsymbol{r} = \begin{pmatrix} -231 \\ -108 \end{pmatrix}\right)$
Specific behaviours
✓ rotates vector 90° ✓ ratio of magnitudes ✓ any correct vector

Question 12 **(8 marks)**

The vertices of triangle T are $A(2,3)$, $B(-5,1)$ and $C(0,12)$.

Transformation M is a translation by vector $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$.

(a) State the coordinates of the image of C after triangle T is transformed by M. (1 mark)

Solution
$\begin{bmatrix} 0 \\ 12 \end{bmatrix} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix} \Rightarrow C'(4,10)$
Specific behaviours
✓ correct coordinates

Transformation N is a reflection in the line $x + y = 0$.

(b) Determine the transformation matrix for N and state the coordinates of the image of A after triangle T is transformed by M and then by N. (3 marks)

Solution
$N = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ $A' = \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$ $A'' = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -6 \end{bmatrix}$ $A''(-1,-6)$
Specific behaviours
✓ matrix for N ✓ transforms A by M ✓ coordinates of A''

Transformation P is a rotation of $135°$ clockwise about the origin.

(c) Determine the exact coordinates of the image of B after triangle T is transformed by N and then by P. (3 marks)

Solution

$$P = \begin{bmatrix} \cos(-135°) & -\sin(-135°) \\ \sin(-135°) & \cos(-135°) \end{bmatrix} = \begin{bmatrix} \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{-\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \end{bmatrix}$$

$$B' = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -5 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

$$B'' = \begin{bmatrix} \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{-\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} -1 \\ 5 \end{bmatrix} = \begin{bmatrix} 3\sqrt{2} \\ -2\sqrt{2} \end{bmatrix}$$

$$B''(3\sqrt{2}, -2\sqrt{2})$$

Specific behaviours

✓ matrix for P
✓ transforms B by N
✓ coordinates of B''

(d) Write a matrix expression for the transformation matrix Q that represents the inverse of transformation P followed by the inverse of transformation N. There is no need to simplify your expression. (1 mark)

Solution

N.B. N^{-1} can be replaced with N below, as N is self inverse.

$$Q = N^{-1} \times P^{-1}$$

Or

$$Q = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}^{-1} \times \begin{bmatrix} \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{-\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \end{bmatrix}^{-1}$$

Or

$$Q = (PN)^{-1}$$

Or

$$Q = \left(\begin{bmatrix} \frac{-\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{-\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \end{bmatrix} \times \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \right)^{-1}$$

Specific behaviours

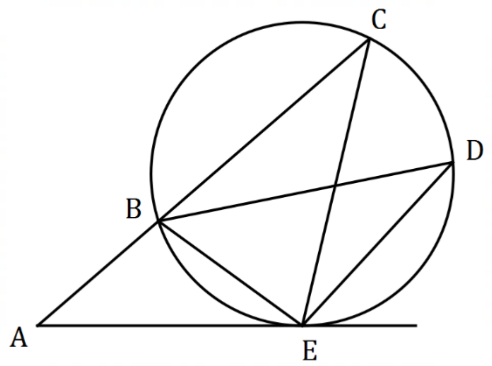
✓ any correct expression

Question 13 **(8 marks)**

(a) In the diagram shown (not to scale) ABC is a straight line and B, C, D and E lie on a circle.

AE is a tangent to the circle at E, $\angle BEC = 76°$ and $\angle BDE = 27°$.

Determine, with reasons, the size of $\angle BAE$.



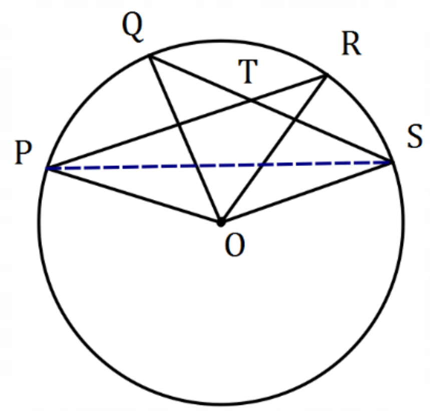
(4 marks)

Solution
$\angle BCE = 27°$ (angles on same arc BE) $\angle BEA = 27°$ (alternate segment theorem) $\angle AEC = 76° + 27° = 103°$ (adjacent angles) $\angle BAE = 180° - 103° - 27° = 50°$ (angle sum in ΔAEC)
Specific behaviours
✓ $\angle BCE$ with reason ✓ $\angle BEA$ with reason ✓ $\angle AEC$ with reason ✓ $\angle BAE$ with reason

(b) In the diagram shown (not to scale) P, Q, R and S lie on a circle centre O and chords QS and PR intersect at T.

$\angle POQ = 42°$ and $\angle ROS = 35°$.

Determine, with reasons, the size of $\angle RTS$.



(4 marks)

Solution
$\angle PSQ = \frac{1}{2} \times 42° = 21°$ (angle at centre-circumference) $\angle RPS = \frac{1}{2} \times 35° = 17.5°$ (angle at centre-circumference) $\angle RTS = 21° + 17.5° = 38.5°$ (sum of opposite interior angles)
Specific behaviours
✓ adds chord PS (or QR) ✓ $\angle PSQ$ with reason ✓ $\angle RPS$ with reason ✓ $\angle RTS$ with reason

Question 14 **(8 marks)**

(a) State whether each of the following statements is true or false, supporting each answer with an example or counterexample.

(i) $\forall a, b, c, d \in \mathbb{R}$, if $a < b$ and $c < d$ then $ac < bd$. (2 marks)

Solution
False. Let $a = -2, b = 1$ and $c = -3, d = 0$. Then $a < b$ and $c < d$ but $ac = 6$ and $bd = 0$ and so $ac > bd$.
Specific behaviours
✓ states false ✓ valid counterexample

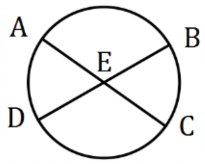
(ii) $\forall n \in \mathbb{N}$, if n is even then $3^n - 2$ is prime. (2 marks)

Solution
False. When $n = 8,\ 3^8 - 2 = 6\,559 = 7 \times 937$ - not prime.
Specific behaviours
✓ states false ✓ valid counterexample using even integer

(b) Prove by contradiction that $ABCD$ is not a cyclic quadrilateral if diagonal AC of length 13 cm cuts diagonal BD of length 12 cm at E so that $AE = DE = 4$ cm. (4 marks)

Solution

Assume that $ABCD$ is a cyclic quadrilateral, as shown below:



$CE = 13 - 4 = 9$ cm and $BE = 12 - 4 = 8$ cm.

By the intersecting chord theorem, $AE \times CE = BE \times DE$

However, $AE \times CE = 4 \times 9 = 36$ but $BE \times DE = 8 \times 4 = 32$ which contradicts our initial assumption and so $ABCD$ is not a cyclic quadrilateral.

Specific behaviours

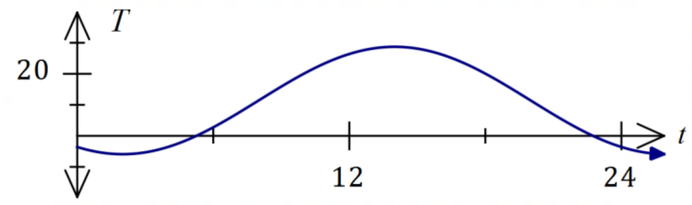
✓ states assumption that quadrilateral is cyclic
✓ calculates correct segment lengths
✓ uses intersecting chord theorem
✓ indicates contradiction

Question 15 **(8 marks)**

Starting at midnight ($t = 0$), the temperature T at a resort was observed to vary sinusoidally over the course of a day, reaching a high of 28.7°C at 2 pm after a low of −5.9°C at 2 am. Let t be the time in hours from midnight.

(a) Use the above information to **sketch** a graph showing how T varies with t during the day. (2 marks)

Solution



Specific behaviours

✓ axes with indication of scale and sinusoidal graph
✓ correctly locates minimum and maximum

(b) Determine an algebraic model for T as a function of t. (4 marks)

Solution (Using cos)

Model with $T = c - a\cos\big(k(t+b)\big)$

Using period: $k = \frac{2\pi}{24} = \frac{\pi}{12}$

Amplitude: $a = \frac{28.7-(-5.9)}{2} = 17.3$

Mean temp: $c = 28.7 - 17.3 = 11.4$

Phase shift: $b = -2$

$$T = 11.4 - 17.3\cos\left(\frac{\pi}{12}(t-2)\right)$$

Specific behaviours

✓ indicates period, value of k
✓ amplitude a and mean c
✓ phase shift
✓ correct model

Solution (Using sin)

Model with $T = a\sin\big(k(t+b)\big) + c$

Using period: $k = \frac{2\pi}{24} = \frac{\pi}{12}$

Amplitude: $a = \frac{28.7-(-5.9)}{2} = 17.3$

Mean temp: $c = 28.7 - 17.3 = 11.4$

Phase shift: $b = -2$

$$T = 17.3\sin\left(\frac{\pi}{12}(t-8)\right) + 11.4$$

Specific behaviours

✓ indicates period, value of k
✓ amplitude a and mean c
✓ phase shift
✓ correct model

(c) Use your model to determine the proportion of the day that the temperature at the resort was below 4°C. (2 marks)

Solution

$T = 4$ when $t = 6.312,\ t = 21.688$

Proportion of day:

$$\frac{(24 - 21.688) + 6.312}{24} = \frac{8.624}{24} \approx 0.36 \text{ or } 36\%$$

Specific behaviours

✓ values of t
✓ correct proportion

Question 16 **(8 marks)**

(a) Determine the number of integers between 1 and 499 that are

(i) divisible by 56. (1 mark)

Solution
$\lfloor 499 \div 56 \rfloor = 8$
Specific behaviours
✓ correct number

(ii) divisible by 7 or by 8 but not by 56. (3 marks)

Solution
Divisible by $7, 8$: $$\lfloor 499 \div 7 \rfloor = 71$$ $$\lfloor 499 \div 8 \rfloor = 62$$ Divisible by 7 or 8: $$71 + 62 - 8 = 125$$ Divisible by 7 or 8 and not 56: $$125 - 8 = 117$$
Specific behaviours
✓ numbers divisible by $7, 8$ ✓ number divisible by 7 or 8 ✓ correct answer

(b) A playlist offered by a music streaming service has 22 different songs. Every time a playlist is streamed, the songs are shuffled into a random arrangement.

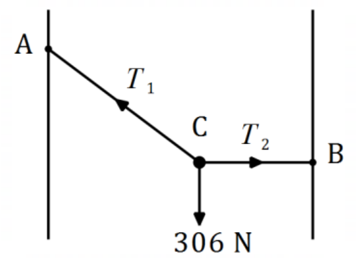
Show that after the playlist has been streamed $30\,000$ times, at least 4 of those streams began with the same 3 songs in the same order. (4 marks)

Solution
Number of different arrangements for first 3 songs: $$^{22}P_3 = 9\,240$$ Using the pigeonhole principle, we have $30\,000$ pigeons to place in $9\,240$ pigeonholes. $$\lceil 30000 \div 9240 \rceil = 4$$ Hence at least 4 of the streams must have begun with the same 3 songs in the same order.
Specific behaviours
✓ number of arrangements ✓ identifies pigeons ✓ identifies pigeonholes ✓ uses pigeonhole principle to draw conclusion

Question 17 **(8 marks)**

A small object C of weight 306 N is suspended above level ground and between two vertical walls by two strings. The walls are 192 cm apart.

Point A lies on one wall so that string AC is 185 cm long and point B lies on the other wall so that string BC is horizontal and 88 cm long.

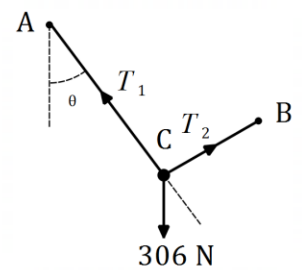


(a) Determine the tension T_1 in string AC. (3 marks)

Solution
Let D be vertically below A so that ADC is a right triangle. Then $DC = 192 - 88 = 104$ cm. $$\angle CAD = \sin^{-1}\frac{104}{185} = 34.2°$$ Resolving vertically: $T_1 \cos 34.2° = 306 \Rightarrow T_1 = 370$ N
Specific behaviours
✓ angle between AC and wall ✓ resolves horizontally ✓ tension T_1

(b) String BC is lengthened so that the height of C above the ground decreases by 29 cm and $\angle ACB = 90°$. Determine the tension T_2 in string BC. (5 marks)

Solution



Resolve ∥ BC: $T_2 = 306 \sin\theta$

$$AD_{OLD} = \sqrt{185^2 - 104^2} = 153$$
$$AD_{NEW} = 153 + 29 = 182$$

$$\cos\theta = \frac{182}{185} \Rightarrow \theta = 10.33°$$

$$T_2 = 306 \sin 10.33° = 54.9 \text{ N}$$

Specific behaviours

✓ resolves parallel to T_2
✓ equation for T_2
✓ determines AD
✓ angle θ
✓ tension T_2

Question 18 **(6 marks)**

(a) Given that $A = \begin{bmatrix} a-3 & 8 \\ 2a+1 & 3-a \end{bmatrix}$, determine the value(s) of the real constant a so that A is its own inverse. (3 marks)

Solution
Require $A^2 = I$:
$$A^2 = \begin{bmatrix} a^2+10a+17 & 0 \\ 0 & a^2+10a+17 \end{bmatrix}$$
$a^2 + 10a + 17 = 1$ $(a+2)(a+8) = 0$ $a = -2, a = -8$
Specific behaviours
✓ indicates that $A^2 = I$ ✓ indicates A^2 ✓ both solutions to $A^2_{1,1} = 1$

(b) Let $B = \begin{bmatrix} -1 & 5 \\ 2 & -8 \end{bmatrix}$ and $C = \begin{bmatrix} 7 \\ -11 \end{bmatrix}$. Determine X when $X - 5BC = B^2X$. (3 marks)

Solution
$X - B^2X = 5BC$ $(I - B^2)X = 5BC$ $X = (I - B^2)^{-1} \times 5BC$
$I - B^2 = \begin{bmatrix} -10 & 45 \\ 18 & -73 \end{bmatrix}$, $(I - B^2)^{-1} = \begin{bmatrix} 73/80 & 9/16 \\ 9/40 & 1/8 \end{bmatrix}$, $5BC = \begin{bmatrix} -310 \\ 510 \end{bmatrix}$
$X = \begin{bmatrix} 4 \\ -6 \end{bmatrix}$
Specific behaviours
✓ indicates (post) factoring of X ✓ indicates correct equation for X ✓ correct matrix X

Question 19 **(8 marks)**

(a) 7 students from Class A, 9 from Class B and 5 from Class C have nominated for the 3 places available in the team for a mathematics competition. Determine the number of different teams that can be formed if

(i) the students are chosen from the same class. (2 marks)

Solution
$\binom{7}{3} + \binom{9}{3} + \binom{5}{3} = 35 + 84 + 10 = 129 \text{ teams}$
Specific behaviours
✓ uses combinations ✓ correct number

(ii) at least 2 students in the team are chosen from Class B. (2 marks)

Solution
$\binom{9}{2}\binom{12}{1} + \binom{9}{3}\binom{12}{0} = 432 + 84 = 516 \text{ teams}$
Specific behaviours
✓ identifies both cases ✓ correct number

(b) Prove that for $n \geq 4$, ${}^{n}C_3 + {}^{n}C_4 = {}^{n+1}C_4$. (4 marks)

Solution
$\begin{aligned} LHS &= {}^{n}C_3 + {}^{n}C_4 \\ &= \frac{n!}{3!\,(n-3)!} + \frac{n!}{4!\,(n-4)!} \\ &= \frac{4 \times n!}{4 \times 3!\,(n-3)!} + \frac{(n-3)n!}{4!\,(n-3)(n-4)!} \\ &= \frac{4n!}{4!\,(n-3)!} + \frac{n \cdot n! - 3n!}{4!\,(n-3)!} \\ &= \frac{n! + n \cdot n!}{4!\,(n-3)!} \\ &= \frac{(n+1)!}{4!\,(n+1-4)!} \\ &= {}^{n+1}C_4 \\ &= RHS \end{aligned}$
Specific behaviours
✓ expresses LHS using factorials ✓ obtains common denominator ✓ simplifies to single fraction ✓ completes proof

Question 20 **(8 marks)**

A common proof that $\sqrt{3}$ is irrational begins by assuming that $\sqrt{3}$ is rational, so that $\sqrt{3} = \frac{a}{b}$.

(a) Describe two properties of variables a and b that the proof requires, other than $b \neq 0$. (2 marks)

Solution
a and b are integers and have no common factor.
Specific behaviours
✓ states both are integers ✓ states no common factor, divisor, etc

The next step obtains the relationship $a^2 = 3b^2$, from which it is deduced that $a = 3A, A \in \mathbb{Z}$.

(b) Prove, using the contrapositive, that if a^2 is a multiple of 3 then so is a. (4 marks)

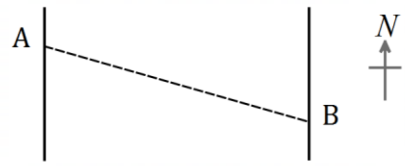
Solution
Contrapositive: If a is not a multiple of 3 then neither is a^2. Note: a must be of the form $3k+1$ or $3k+2$, $k \in \mathbb{Z}$ so that it is 1 or 2 more than an integer multiple of 3. Case 1: $a = 3k+1 \Rightarrow a^2 = 9k^2 + 6k + 1 = 3(3k^2 + 2k) + 1$ Case 2: $a = 3k+2 \Rightarrow a^2 = 9k^2 + 6k + 4 = 3(3k^2 + 2k + 1) + 1$ It can be seen in each case that a^2 is not an integer multiple of 3. As the contrapositive is true then the original statement must be true.
Specific behaviours
✓ writes contrapositive ✓ identifies cases for a in terms of some constant integer ✓ shows a^2 is not multiple of 3 for one case ✓ shows a^2 is not multiple of 3 for other case and concludes

(c) Complete the proof that $\sqrt{3}$ is irrational. (2 marks)

Solution
Since $a = 3A$ then $a^2 = 3b^2 \Rightarrow (3A)^2 = 3b^2 \Rightarrow b^2 = 3A^2$. Thus b^2 and b are also multiples of 3. Hence a and b are both multiples of 3 - a contradiction of the initial assumption and so $\sqrt{3}$ is irrational.
Specific behaviours
✓ deduces that b is multiple of 3 ✓ indicates contradiction

Question 21 **(8 marks)**

Points A and B lie on opposite sides of a river so that B is 240 m away from A on a bearing of 105°.

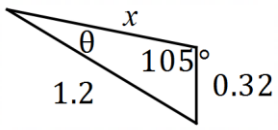


A uniform current flows due north in the river between A and B at 0.32 m/s.

Sam can swim at a steady speed of 1.2 m/s and plans to swim from A to B and then back to A.

(a) Determine the bearing Sam should swim to move directly towards B from A. (3 marks)

Solution



$$\frac{\sin\theta}{0.32} = \frac{\sin 105°}{1.2} \Rightarrow \theta = 14.9°$$

Bearing: $105° + 15° = 120°$

Specific behaviours

✓ diagram with angle
✓ equation using sine rule
✓ correct bearing

(b) Show that Sam takes 30 seconds less to swim the return leg than the first leg. (5 marks)

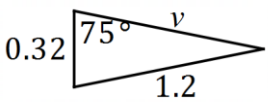
Solution

Speed across ground from A to B:

$$\frac{\sin(180° - 105° - 14.9°)}{x} = \frac{\sin 105°}{1.2} \Rightarrow x = 1.077 \text{ m/s}$$

Time $AB = 240 \div 1.077 = 223$ s

Return leg from B to A:



$$1.2^2 = v^2 + 0.32^2 - 2(0.32)v\cos 75° \Rightarrow v = 1.242 \text{ m/s}$$

Time $BA = 240 \div 1.242 = 193$ s

Hence $223 - 193 = 30$ second less.

Specific behaviours

✓ speed from A to B
✓ time from A to B
✓ diagram for B to A
✓ speed from B to A
✓ time from B to A and difference

End of questions

Supplementary page

Question number: _________

Supplementary page

Question number: _________

Supplementary page

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